Part 10: Linearised Rasch Models (II)

repeated measurements

LLTM (2) - Repeated measurements

research question: does individual's test performance changes over time?

most intuitive way: look at shift in ability θ_v across time points

alternative: assume person parameters fixed over time
look for change of item parameters

Concept of virtual items

basic idea:
item i is presented at two different times t_1 and t_2 to the same person v
this is regarded as a pair of *virtual items*
change in θ_v can be described as change of item parameters

Concept of virtual items(cont'd)

for two measurement points t_1 and t_2:

item i with the corresponding parameter β_i generates two virtual items item a and b with associated item parameters β_a^* and β_b^*

for the first measurement point: $\beta_a^* = \beta_i$
for the second: $\beta_b^* = \beta_i + \boldsymbol{\tau}$

in this linear combination the $\boldsymbol{\beta}^*$-parameters are composed additively by means of the real item parameters $\boldsymbol{\beta}$ and the effects τ (treatment, general trend, etc.)

this concept extends to an arbitrary number of time points or testing occasions

LLTM (2) - Design Matrix for 2 Time Points

		η_1	η_2	$\cdots$	η_k	η_{k+1}
Time 1	$\beta_1^{*(1)}$	1	0	0	0	0
	$\beta_2^{*(1)}$	0	1	0	0	0
	$\vdots$			$\ddots$		$\vdots$
	$\beta_k^{*(1)}$	1	0	0	1	0
Time 2	$\beta_{k+1}^{*(2)}$	1	0	0	0	1
	$\beta_{k+2}^{*(2)}$	0	1	0	0	1
	$\vdots$			$\ddots$		$\vdots$
	$\beta_{2k}^{*(2)}$	1	0	0	1	1

$\boldsymbol{\beta}^{*(1)}$, $\boldsymbol{\beta}^{*(2)}$ . . . item parameters for the two time points
η_{k+1} here describes a constant shift for all item parameters
η_1 cannot be estimated, restriction here $\eta_1 = 0$
for a more general setting see Fischer & Molenaar (1995, p.159)

LLTM (2) - in eRm

this is a reduced example from the eRm help file for `LLTM()`
2 measurement (time) points, we use only 3 items here

```
> data(lltmdat1)
> dat <- lltmdat1[, c(1:3, 16:18)]
> res1 <- LLTM(dat, mpoints = 2)
> res1
Results of LLTM estimation:

Call:  LLTM(X = dat, mpoints = 2)

Conditional log-likelihood: -211
Number of iterations: 5
Number of parameters: 3

Basic Parameters eta:
         eta 1 eta 2 eta 3
Estimate -0.13 0.016 -0.85
Std.Err   0.13 0.127  0.18
```



```
> summary(res1)
Results of LLTM estimation:

Call:  LLTM(X = dat, mpoints = 2)

Conditional log-likelihood: -211
Number of iterations: 5
Number of parameters: 3

Basic Parameters eta with 0.95 CI:
      Estimate Std. Error lower CI upper CI
eta 1   -0.128       0.13    -0.38     0.12
eta 2    0.016       0.13    -0.23     0.26
eta 3   -0.847       0.18    -1.20    -0.49

Item Easiness Parameters (beta) with 0.95 CI:
      Estimate Std. Error lower CI upper CI
I1 t1    0.112       0.13    -0.14     0.36
I2 t1   -0.128       0.13    -0.38     0.12
I3 t1    0.016       0.13    -0.23     0.26
I1 t2   -0.735       0.22    -1.17    -0.30
I2 t2   -0.975       0.22    -1.41    -0.54
I3 t2   -0.831       0.22    -1.26    -0.40
```


LLTM (2) - in eRm

the 'autogenerated' design' matrix is

```
> model.matrix(res1)
      eta 1 eta 2 eta 3
I1 t1    -1    -1     0
I2 t1     1     0     0
I3 t1     0     1     0
I1 t2    -1    -1     1
I2 t2     1     0     1
I3 t2     0     1     1
```

remarks:
rows 1 and 4 indicate sum-zero constraints: $\sum \beta_i^{*(1)} = \sum \beta_i^{*(2)} = 0$
eta 1 $= \eta_2$ . . . 'common' itemparameter for item 2
eta 2 $= \eta_3$ . . . 'common' itemparameter for item 3
eta 3 $= \eta_{k+1}$ . . . 'Trend' parameter (shift from t_1 to t_2)

LLTM (2) - Repeated measurements

in practical research:

dependent on research question:

- if measurement principles are relevant:
Rasch homogeneity should be checked for t_1 and t_2 separately
after establishing a common scale (the same valid items for both time points) the LLTM is fitted

shift can be modelled:
e.g., different 'shift' parameters for different (groups of) items, treatment groups

extension to more time points straightforward

- if measuring change is of primary interest:
use other model $\longrightarrow$ LLRA

Linear Partial Credit Model (LPCM)

the principles of the LLTM also apply to polytomous responses

the LPCM (Fischer & Ponocny, 1994) decomposes the item × category parameters β_{ih}

$$P(X_{vih} = 1) = \frac{\exp(h\theta_v + \beta_{ih})}{\sum_{l=0}^{m_i} \exp(l\theta_v + \beta_{il})},$$

linearly with

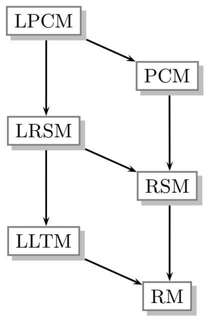
$$\beta_{ih} = \sum_{j=1}^{p} w_{ihj}\eta_j$$

$\eta_j,\ j = 1, \ldots, p$ are the 'basic' parameters (as before)
w_{ihj} are the weights of η_j for β_{ih} ($w_{i0j} = 0\ \forall\ i, j$).

the Linear Rating Scale Model (LRSM, Fischer & Parzer) is a special case of the LPCM like the RSM of the PCM

The model hierarchy in eRm

the PCM is the most general unidimensional model in this family
all other models are submodels
they are obtained by appropriately defining the design matrix W



Design for a PCM

Example: 3 Items – number of categories: $m_1 = 3$, $m_2 = 4$, $m_3 = 2$

	eta 1	eta 2	eta 3	eta 4	eta 5
beta I1.c1	0	0	0	0	0
beta I1.c2	1	0	0	0	0
beta I2.c1	0	1	0	0	0
beta I2.c2	0	0	1	0	0
beta I2.c3	0	0	0	1	0
beta I3.c1	0	0	0	0	1

here we show 'treatment' constraints
(for sum-zero constraints first row consists of -1's)

all betas in category 0 are set to 0, i.e., $\beta_{i0} = 0$ for all i
additionally `beta I1.c1` is set to 0, i.e., $\beta_{11} = 0$

`eta 1` corresponds to $\beta_{ih} = \beta_{12}$, `eta 2` corresponds to β_{21}, etc.

Design for a RM

Example: 4 Items – number of categories: $m_i = 2$

	eta 1	eta 2	eta 3
beta I1.c1	0	0	0
beta I2.c1	1	0	0
beta I3.c1	0	1	0
beta I4.c1	0	0	1

as for the PCM: (but now only 2 categories)
all betas in category 0 are set to 0, i.e., $\beta_{i0} = 0$ for all i
additionally `beta I1.c1` is set to 0, i.e., $\beta_{11} = 0$

specification of categories in case of the RM superfluous

`eta 1` corresponds to β_2, `eta 2` corresponds to β_3, etc.

Design for a RSM

Example: 3 Items – number of categories: $m_i = 3$

```
           eta 1 eta 2 eta 3
beta I1.c1     0     0     0
beta I1.c2     0     0     1
beta I2.c1     1     0     0
beta I2.c2     2     0     1
beta I3.c1     0     1     0
beta I3.c2     0     2     1
```

the linear predictor of the RSM is: $h(\theta_v + \beta_i) + \omega_h$

the first beta is set to 0, i.e., $\beta_1 = 0$
the first two category parameters are set to 0, i.e., $\omega_0 = \omega_1 = 0$

eta 1 corresponds to β_2, eta 2 corresponds to β_3
eta 3 corresponds to ω_2

for 4 categories: there is an eta 4 corresponding to ω_3

Design for a LLTM

Example: 3 Items, 2 time points, 2 treatments

```
         eta 1 eta 2 eta 3 eta 4
I1 t1 g1     0     0     0     0
I2 t1 g1     1     0     0     0
I3 t1 g1     0     1     0     0
I1 t1 g2     0     0     0     0
I2 t1 g2     1     0     0     0
I3 t1 g2     0     1     0     0
I1 t2 g1     0     0     1     0
I2 t2 g1     1     0     1     0
I3 t2 g1     0     1     1     0
I1 t2 g2     0     0     1     1
I2 t2 g2     1     0     1     1
I3 t2 g2     0     1     1     1
```

I1 t1 g1 is β for Item 1 at t_1 in treatment group g1
eta 1 and eta 2 correspond to β_2 and β_3
eta 3 is the time effect. eta 4 is the treatment effect

R commands

main functions concerning fit of the 'L' models:

- `LPCM(X, W, mpoints, groupvec, ... )` fits the LPCM
- `LRSM(X, W, mpoints, groupvec, ... )` fits the LRSM
- `LTTM(X, W, mpoints, groupvec, ... )` fits the LTTM
- all other functions are the same as previously presented (not functions that are restricted to `Rm` and `dRm` objects)

– `LPCM()`, `LRSM()`, `LLTM()` generate objects of class `eRm`
– `mpoints = t` is for data with `t` timepoints
– `groupvec = p` is for data with `p` groups
– if `mpoints` or `groupvec` is specified, a simple design is generated
– for non-standard design the design matrices `W` has to be supplied

for details see the help for these functions