Part 9: Linearised Rasch Models - The restricted LLTM

The 'L' Models:

Linear Logistic Test Model (LLTM)
Linear Partial Credit Model (LPCM)
Linear Rating Scale Model (LRSM)
Linear Logistic Model with Relaxed Assumptions (LLRA)

Linear Logistic Test Model (LLTM)

LLTM is a linearised version of the RM

item parameters β_i, $(i = 1, \ldots, k)$ are decomposed into a weighted sum of so called "basic parameters", η_j, $(j = 1, \ldots, p)$

$$P(X_{vi} = 1) = \frac{\exp(\theta_v - \beta_i)}{1 + \exp(\theta_v - \beta_i)}$$

with

$$\beta_i = \sum_{j=1}^{p} w_{ij}\eta_j \qquad i = 1, \ldots, k.$$

η_j are the basic parameters of the LLTM
w_{ij} are fixed weights ('item-specific covariates')

LLTM

general idea: reparameterisation of the item parameters

generalises the RM in two ways:

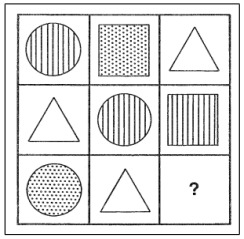
- Item Structure Model (1) (Scheiblechner, 1972)
  (more restrictive than RM)
  – evaluate components of items
  – construction of adaptive tests
  – taylored testing

- Repeated Measurement Model (2) (Fischer, 1972)
  (extending the RM)
  – measuring change on latent dimension
  – specification of groups possible
  – experimental designs

LLTM (1) - Item Structure Model

Items are composed of different components
('item specific' covariates)

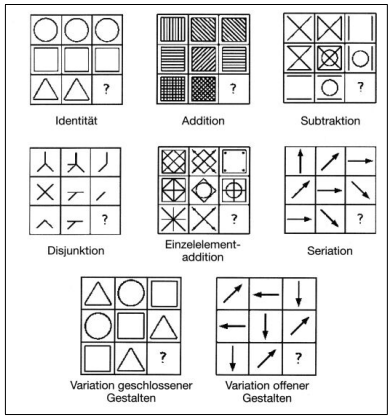
Example: Item Bank Construction (Hornke & Habon, 1986)
in the context of research on cognitive operations



Item is composed of
3 structural components

difficulty of the components makes up the difficulty of the item

LLTM (1) - Test construction



Components are:

- Operations (see left)
- Relations:
  - by row
  - by column
  - both

  (see the upper three items)
- these two may be
  - separated
  - integrated
  - embedded

LLTM (1) - Test construction

the designmatrix could be:

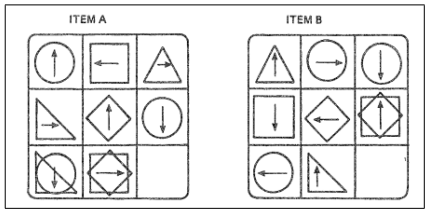
	η_1	η_2	$\cdots$	η_p
β_1	1	0	$\cdots$	0
β_2	0	1	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
β_k	1	0	$\cdots$	1

β_i represent the items
$\eta_1, \ldots, \eta_p$ represent the components which make up the items

two models are estimated: RM($\boldsymbol{\beta}$) and LLTM($\boldsymbol{\beta}$)
first the RM evaluated using various test procedures
if the RM holds, the correponding LLTM is estimated and tested against the RM

LLTM (1) - Test construction

Item Difficulties calculated from Operation Difficulty Estimates



$\widehat{\beta_A} = \text{SC} + \text{IS} + \text{SE} + 2\text{C} = -0.51 + 0.76 - 0.20 + 0.08 \cdot 2 = 0.21$
$\widehat{\beta_B} = \text{SC} + \text{IS} + \text{SE} + 2\text{C} = -0.51 + 0.76 - 0.20 - 0.23 \cdot 2 = -0.41$

can be used to predict item difficulties for individual testing

LLTM (1) - in eRm

this is an example from the eRm help file for `LLTM()`

```
> data(lltmdat2)
> W <- matrix(c(1, 2, 1, 3, 2, 2, 2, 1, 1, 1), ncol = 2)
> rownames(W) <- paste("Item", 1:nrow(W), sep = "")
> colnames(W) <- c("CovA", "CovB")
```

W is a design matrix, here the entries are numerical

```
> W
      CovA CovB
Item1    1    2
Item2    2    2
Item3    1    1
Item4    3    1
Item5    2    1
```

LLTM (1) - in eRm

fit a RM and an LLTM

```
> res1 <- RM(lltmdat2)
> res2 <- LLTM(lltmdat2, W = W)
> print(res2)
Results of LLTM estimation:

Call:  LLTM(X = lltmdat2, W = W)

Conditional log-likelihood: -31.65225
Number of iterations: 7
Number of parameters: 2

Basic Parameters eta:
                CovA      CovB
Estimate -0.09775528 0.1141153
Std.Err   0.31296155 0.4779270
```


LLTM (1) - in eRm

assuming the RM holds, is the reparameterisation permissable?

```
> devdiff <- 2 * (res1$loglik - res2$loglik)
> dfdiff <- res1$npar - res2$npar
> pval <- 1 - pchisq(devdiff, dfdiff)
> cat("Deviance Difference: ", devdiff, "df: ", dfdiff, "p-value: ",
+     pval, "\n")
Deviance Difference:  1.048847 df:  2 p-value:  0.5918966
```